Seminary 6
Periodical motion. Oscillations

SIMPLE HARMONIC OSCILLATOR

1/ If a pendulum clock is taken to a mountaintop, does it gain or lose time, assuming it is correct at a lower elevation? Explain your answer.

2/ A 1.50-kg mass on a spring has displacement as a function of time given by the equation:

$$x(t) = (7.40 \text{ cm}) \cos[(4.16 \text{ s}^{-1})t - 2.42]$$

Find (a) the time for one complete vibration; (b) the force constant of the spring; (c) the maximum speed of the mass; (d) the maximum force on the mass; (e) the position, speed, and acceleration of the mass at t=1s (f) the force on the mass at that time.

3/ A 0.500-kg mass on a spring has velocity as a function of time given by

$$v_x(t) = -(3.60 \text{ cm/s}) \sin[(4.71 \text{ s}^{-1})t - \pi/2].$$

What are (a) the period; (b) the amplitude; (c) the maximum acceleration of the mass; (d) the force constant of the spring?

4/ The equation of movement for a simple harmonic oscillator with mass m=0.1 kg is: $y(t) = 2\sin\left(4t + \frac{\pi}{2}\right)$ [m]. Calculate the kinetic, the potential and the total energy of the oscillator. Discuss the result.

5/ A harmonic oscillator has angular frequency ω and amplitude A. (a) What are the magnitudes of the displacement and velocity when the elastic potential energy is equal to the kinetic energy? (Assume that at U=0 equilibrium.) (b) How often does this occur in each cycle? What is the time between occurrences? (c) At an instant when the displacement is equal to what fraction of the total energy of the system is kinetic and what fraction is potential?

6/ A mass is oscillating with amplitude *A* at the end of a spring. How far (in terms of *A*) is this mass from the equilibrium position of the spring when the elastic potential energy equals the kinetic energy?

DAMPED OSCILLATOR

1/ The motion of an underdamped oscillator is described by

$x = Ae^{-(b/2m)t}\cos(\omega' t + \phi)$ with : $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$ Let the phase angle ϕ be zero. (a) According to this equation, what is the value of x at t=0? (b) What are the magnitude and direction of the velocity at t=0? What does the result tell you about the slope of the graph of x versus t near t=0? (c) Obtain an expression for the acceleration a_x at t=0. For what value or range of values of the damping constant b (in terms of k and m) is the acceleration at negative, zero, and positive? Discuss each case in terms of the shape of the graph of x versus t near t = 0.

DRIVEN OSCILLATOR AND RESONANCE

1/ A drilling machine was found to vibrate so much that accurate work could not be done at certain frequencies. An investigation of its behaviour showed that the amplitude of the vibration of the drill *A* was related to the frequency of rotation as follows:

A / 10–2 mm	**f / Hz**
0	0
14	5
30	8
44	9
80	10
96	11
24	12
8	13
2	15
3	20
9	25
7	30
4	35

(a) Draw a graph of *A* against *f* and explain its shape. (b) Why is it advisable to start to drill a hole with the drill rotating at a frequency of between 15 Hz and 20 Hz?

2/ A sinusoidally varying driving force is applied to a damped harmonic oscillator of force constant *k* and mass *m*. If the damping constant has a value b_1 the amplitude is A_1 when the driving angular frequency equals $\sqrt{k/m}$. In terms of A_1, what is the amplitude for the same driving frequency and the same driving force amplitude F_{max} if the damping constant is (a) $3b_1$ and (b) $b_1/2$?

Given:

$$A = \frac{F_{\max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}$$

Discussion: Explain the origin of Ocean tides

The gravitational attraction of the moon causes the ocean tides. This gravitational force is constant. However, some areas experience higher tides than others. The answer lies in the study of resonance. Bays of certain shape oscillate naturally, as waves hit the shore, travel toward the center of the bay, then deflect back to the shore. The moon, then, can be seen as a driving force, which varies sinusoidally as it rotates about the earth. Thus, if the natural frequency of the bay and the frequency of the driving force are similar, the amplitude of oscillation (the size of the tide) will increase greatly. In some places the two frequencies are quite different, resulting in little change in the tide.